Math 60 7.5 Adding and Subtracting Rational Expressions With Unlike Denominators

Objectives:

1) Adding and Subtracting Rational Expressions With Unlike Denominators
   a. Find the Least Common Denominator (LCD) of two rational expressions
   b. Use the LCD to write equivalent rational expressions.
   c. Combine like terms in numerators
   d. Leave LCD unchanged
   e. Last: Factor numerator and cancel and common factors
   f. Leave final answer factored

Examples and Practice:

Perform the indicated operations and simplify. (Use the order of operations, if appropriate!)

1) $\frac{1}{6}+\frac{9}{14}$

2) $\frac{5}{6x^2}+\frac{4}{15x}$

3) $\frac{-2}{x+5}+\frac{3}{x+4}$

4) $\frac{3}{x+2}+\frac{8-2x}{x^2+4}$

5) $\frac{3}{x+2}+\frac{8-2x}{x^2-4}$

6) $\frac{7}{2a}-\frac{3}{a+5}$

7) $\frac{4}{x^2-6x}+\frac{5}{2x-12}$

8) $\frac{2y+8}{y^2+4y-12}-\frac{y+1}{y^2-2y}$

9) $\frac{2}{3-b}+\frac{1}{b^2-9}$

10) $2+\frac{3}{2x-5}$

11) $\frac{3}{x+2}+\frac{2}{2-x}-\frac{5}{x^2-4}$

12) $\frac{a+3}{a-3}-\frac{a+3}{a-3}\cdot\frac{a^2-4a+3}{a^2+5a+6}$

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① $\frac{1}{6} + \frac{9}{14}$

factor $6 = 2 \cdot 3$
factor $14 = 2 \cdot 7$
LCD $= 2 \cdot 3 \cdot 7 = 42$

$\frac{1}{6} \cdot \frac{7}{7} = \frac{7}{42}$

$\frac{9}{14} \cdot \frac{3}{3} = \frac{27}{42}$

} same work as 7.4
find LCD
write equivalent rational expressions using LCD

$\frac{7}{42} + \frac{27}{42}$ add numerators

$= \frac{34}{42}$ keep common denominator

factor $34 = 2 \cdot 17$
factor $42 = 2 \cdot 3 \cdot 7$

$\frac{34}{42} = \frac{2 \cdot 17}{2 \cdot 3 \cdot 7} = \boxed{\frac{17}{21}}$

} simplify final answer by factor and cancel.

② $\frac{5}{6x^2} + \frac{4}{15x}$

factor $6x^2 = 2 \cdot 3 \cdot x \cdot x$
factor $15x = 3 \cdot 5 \cdot x$
LCD $= 2 \cdot 3 \cdot 5 \cdot x \cdot x = 30x^2$

$\frac{5}{6x^2} \cdot \frac{5}{5} = \frac{25}{30x^2}$

$\frac{4}{15x} \cdot \frac{2x}{2x} = \frac{8x}{30x^2}$

} Find and write with LCD

$\frac{25}{30x^2} + \frac{8x}{30x^2}$

$= \boxed{\frac{8x+25}{30x^2}}$

③ $\frac{-2}{x+5} + \frac{3}{x+4}$ LCD $= (x+5)(x+4)$

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$$\frac{-2(x+4)}{(x+5)(x+4)} + \frac{3(x+5)}{(x+4)(x+5)}$$

multiply each fraction by 1 = $\frac{\text{missing factor}}{\text{missing factor}}$

$$= \frac{-2x-8+3x+15}{(x+4)(x+5)}$$

distribute

$$= \boxed{\frac{x+7}{(x+4)(x+5)}}$$

combine like terms

④ $\frac{3}{x+2} + \frac{8-2x}{x^2+4}$

$(x+2)$ and (x^2+4) are prime

sum of squares a^2+b^2 is always prime!

$LCD = (x+2)(x^2+4)$

$$\frac{3}{(x+2)}\cdot\frac{(x^2+4)}{(x^2+4)} + \frac{(8-2x)}{(x^2+4)}\frac{(x+2)}{(x+2)}$$

multiply each fraction by 1

NOTE: Add parentheses

$$= \frac{3x^2+12+8x+16-2x^2-4x}{(x+2)(x^2+4)}$$

distribute $3(x^2+4)$

FOIL $(8-2x)(x+2)$

$$= \boxed{\frac{x^2+4x+28}{(x+2)(x^2+4)}}$$

28: 1×28, 2×14, 4×7

4

prime

⑤ $\frac{3}{x+2} + \frac{8-2x}{x^2-4}$

$(x+2)$

$x^2-4 = (x+2)(x-2)$

$LCD = (x+2)(x-2)$

} factor + find LCD

$$\frac{3}{(x+2)}\cdot\frac{(x-2)}{(x-2)} + \frac{8-2x}{(x+2)(x-2)}$$

} write with LCD

$$= \frac{3x-6+8-2x}{(x+2)(x-2)}$$

} distribute 3 then combine like terms

$$= \frac{(x+2)}{(x+2)(x-2)} = \boxed{\frac{1}{x-2}}$$

cancel common factor LAST.

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⑥ $\frac{7}{2a} - \frac{3}{a+5}$

$2a = 2 \cdot a$
$(a+5)$
$LCD = 2a(a+5)$
} factor + find LCD

$\frac{7}{2a} \cdot \frac{(a+5)}{(a+5)} - \frac{3}{(a+5)} \cdot \frac{2a}{2a}$
} write equivalent fractions

$= \frac{7a+35}{2a(a+5)} - \frac{6a}{2a(a+5)}$
} dist $7(a+5)$

$= \frac{7a+35-6a}{2a(a+5)}$ ← subtract numerator

$= \boxed{\frac{a+35}{2a(a+5)}}$

⑦ $\frac{4}{x^2-6x} + \frac{5}{2x-12}$

factor $x^2-6x = x(x-6)$
factor $2x-12 = 2(x-6)$
$LCD = 2x(x-6)$
} factor + find LCD

$\frac{4}{x(x-6)} \cdot \frac{2}{2} + \frac{5}{2(x-6)} \cdot \frac{x}{x}$
} write equivalent fractions

$= \frac{8}{2x(x-6)} + \frac{5x}{2x(x-6)}$
} mult $4 \cdot 2$ and $5 \cdot x$

$= \boxed{\frac{5x+8}{2x(x-6)}}$
} numerator not like terms, can't combine. (can't factor + cancel either)

⑧ $\frac{2y+8}{y^2+4y-12} - \frac{y+1}{y^2-2y}$

factor $y^2+4y-12$
$(y-2)(y+6)$

-12
$-2 \quad 6$
4

-1×12
-2×6
-3×4

cont →

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factor $y^2-2y = y(y-2)$ GCF

LCD $y(y-2)(y+6)$

$$\frac{(2y+8)}{(y-2)(y+6)} \cdot \frac{y}{y} - \frac{(y+1)}{y(y-2)} \cdot \frac{(y+6)}{(y+6)}$$

write equivalent fractions

$$= \frac{2y^2+8y}{y(y-2)(y+6)} - \frac{y^2+7y+6}{y(y-2)(y+6)}$$

dist $y(2y+8)$

FOIL $(y+1)(y+6)$

$$= \frac{2y^2+8y-(y^2+7y+6)}{y(y-2)(y+6)}$$

VERY IMPORTANT: Use parentheses and distribute negative to all terms in the second numerator

$$= \frac{2y^2+8y-y^2-7y-6}{y(y-2)(y+6)}$$

$$= \frac{y^2+y-6}{y(y-2)(y+6)}$$

combine like terms

y^2+y-6

$(y+3)(y-2)$ (−6: 3, −2: 1)

$$= \frac{(y+3)(y-2)}{y(y-2)(y+6)}$$

factor and cancel

$$= \boxed{\frac{y+3}{y(y+6)}}$$

(9) $\frac{2}{3-b} + \frac{1}{b^2-9}$

$3-b = -b+3 = -(b-3) \Rightarrow \frac{2}{3-b} = \frac{-2}{b-3}$

$b^2-9 = (b+3)(b-3)$

LCD $= (b+3)(b-3)$

$$\frac{-2}{(b-3)} \cdot \frac{(b+3)}{(b+3)} = \frac{-2b-6}{(b-3)(b+3)}$$

cont →

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$$\frac{-2b-6}{(b-3)(b+3)} + \frac{1}{(b-3)(b+3)}$$

dist $-2(b+3)$

$$= \frac{-2b-6+1}{(b-3)(b+3)}$$

add numerators

combine like terms

$$= \boxed{\frac{-2b-5}{(b-3)(b+3)}}$$

Both are correct.

$$= \boxed{\frac{-(2b+5)}{(b-3)(b+3)}}$$

I slightly prefer this.

(10) $2 + \frac{3}{2x-5}$

$2 = \frac{2}{1}$ so LCD $= (2x-5)$

$$\frac{2}{1}\cdot\frac{(2x-5)}{(2x-5)} + \frac{3}{(2x-5)}$$ ← write equivalent fractions

$$= \frac{4x-10+3}{2x-5}$$ ← dist $2(2x-5)$, add numerators

$$= \boxed{\frac{4x-7}{2x-5}}$$ ← combine like terms

(11) $\frac{3}{x+2} + \frac{2}{2-x} - \frac{5}{x^2-4}$

↑ add ↑ subtract

order of operations: add and subtract from left to right

Method 1: Two separate steps

$$\frac{3}{x+2} + \frac{2}{2-x}$$

$(x+2)$ already factored

$(2-x) = -(x-2)$ so $\frac{2}{2-x} = \frac{-2}{x-2}$

LCD $= (x+2)(x-2)$

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$$\frac{3}{(x+2)}\frac{(x-2)}{(x-2)} + \frac{-2}{(x-2)}\frac{(x+2)}{(x+2)}$$ ← write with LCD

$$= \frac{3x-6}{(x+2)(x-2)} + \frac{-2x-4}{(x-2)(x+2)}$$ ← dist $3(x-2)$, dist $-2(x+2)$

$$= \frac{3x-6-2x-4}{(x-2)(x+2)}$$ ← add numerators

$$= \frac{x-10}{(x-2)(x+2)}$$ ← combine like terms (It won't factor + cancel.)

Now subtract:

$$\frac{x-10}{(x-2)(x+2)} - \frac{5}{(x^2-4)}$$

factor $x^2-4 = (x+2)(x-2)$ ← we already have a common denominator

$$\frac{x-10-5}{(x-2)(x+2)}$$

$$= \boxed{\frac{x-15}{(x-2)(x+2)}}$$

Method 2: All at once

$$\frac{3}{(x+2)}\cdot\frac{(x-2)}{(x-2)} + \frac{-2(x+2)}{(x-2)(x+2)} - \frac{5}{(x-2)(x+2)}$$

$$= \frac{3x-6-2x-4-5}{(x+2)(x-2)}$$ ← distribute $3(x-2)$, $-2(x+2)$; add numerators, subtract numerator

$$= \boxed{\frac{x-15}{(x+2)(x-2)}}$$

(12) $\frac{a+3}{a-3} - \frac{a+3}{a-3} \cdot \frac{a^2-4a+3}{a^2+5a+6}$ ↑ Subtract ↑ multiply

ORDER OF OPERATIONS!
multiply before subtract!

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$$\frac{a+3}{a-3} - \underbrace{\frac{(a+3)}{(a-3)} \cdot \frac{(a-3)(a-1)}{(a+3)(a+2)}}$$

multiply means factor and cancel

a^2-4a+3 (3, −1; −3, −4)
$=(a-3)(a-1)$

a^2+5a+6 (6; 3, 2; 5)
$=(a+3)(a+2)$

$$= \frac{a+3}{a-3} - \frac{a-1}{a+2}$$

write with LCD

$LCD = (a-3)(a+2)$

$$= \frac{(a+3)(a+2)}{(a-3)(a+2)} - \frac{(a-1)(a-3)}{(a+2)(a-3)}$$

FOIL numerators

$$= \frac{(a^2+5a+6)}{(a-3)(a+2)} - \frac{(a^2-4a+3)}{(a-3)(a+2)}$$

$$= \frac{a^2+5a+6-(a^2-4a+3)}{(a-3)(a+2)}$$

← Subtract numerators use parentheses to dist neg to all terms!

$$= \frac{a^2+5a+6-a^2+4a-3}{(a-3)(a+2)}$$

$$= \frac{9a+3}{(a-3)(a+2)}$$

$$= \boxed{\frac{3(3a+1)}{(a-3)(a+2)}}$$